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LETTER TO THE EDITOR

Inverse scattering equations for $g\phi^3+h\phi^4$ theory in two dimensions

A Roy Chowdhury

High Energy Physics Division, Department of Physics, Jadavpur University, Calcutta-700 032, India

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Abstract. Linear equations which can be used for inverse scattering solutions for solitons in $g\phi^3+h\phi^4$ theory have been determined.

In recent times there have been vigorous attempts to obtain Lax type (1968) equations associated with nonlinear partial differential equations, for obtaining soliton-like solutions through the inverse scattering technique. However it was not possible to obtain such a linear system for the theory in two dimensions, which served as a testing ground foı many theoretical formulations. But one, and many soliton-like solutions, could be obtained by the direct technique of Hirota. Here we report a set of linear equations whose consistency condition yields the Klein–Gordon equation with polynomial nonlinearity.

Let e_μ be a set of two-dimensional vectors ($\mu=1,2$) such that $(e_\mu \cdot e_\nu)=g_{\mu\nu}$; then we set

$$L=\begin{pmatrix}(\lambda/2)\phi^2 & \mathrm{i}\sqrt{\lambda/2}\sum e_\mu\phi_\mu \\ -\mathrm{i}\sqrt{\lambda/2}\sum e_\mu\phi_\mu & -(\lambda/2)\phi^2\end{pmatrix}+\frac{1}{2}\begin{pmatrix}a\phi+b & 0 \\ 0 & -a\phi-b\end{pmatrix}$$

$$A=\sqrt{\lambda/2}\begin{pmatrix}0 & \phi \\ \phi & 0\end{pmatrix}. \tag{1}$$

It is then easily seen that the compatibility condition

$$\sum e_\mu(\partial/\partial x_\mu)L+[A,L]=0 \tag{2}$$

is equivalent to the Klein–Gordon equation

$$\Box^2\phi=\lambda\phi^3+a\phi^2+b\phi.$$

The detailed analysis of these linear systems is at present under study and will be communicated shortly.

References

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